Flat directions in the electric theory

For $N_f \geq N_c + 2$:

$$Q = \begin{pmatrix} a_1 & & & \\ & a_2 & & 0 \\ & & \ddots & \\ & & & a_{N_c} \\ & 0 & & \end{pmatrix} ; \quad \tilde{Q} = \begin{pmatrix} \tilde{a}_1 & & & \\ & \tilde{a}_2 & & 0 \\ & & \ddots & \\ & & & \tilde{a}_{N_c} \\ & 0 & & \end{pmatrix}$$

with $|a_i|^2 - |\tilde{a}_i|^2 = \text{const.}$ (independent of i)

Gaug invariant description is given in terms of constrained observables M, B and $\tilde{B}$.

$$M = \begin{pmatrix} a_1\tilde{a}_1 & & & \\ & a_2\tilde{a}_2 & & 0 \\ & & \ddots & \\ & & & a_{N_c}\tilde{a}_{N_c} \\ & 0 & & \end{pmatrix}$$

$$B^{1,\dots,N_c} = a_1 a_2 \cdots a_{N_c}$$

$$\tilde{B}_{1,\dots,N_c} = \tilde{a}_1 \tilde{a}_2 \cdots \tilde{a}_{N_c}$$

with all other components of $M, B, \tilde{B}$ vanishing.

$\longrightarrow \operatorname{rank} M \leq N_c$

if $\operatorname{rank} M < N_c$: $B = 0$ or $\tilde{B} = 0$

$(\operatorname{rk} \tilde{B} \leq 1)$ $\quad$ $(\operatorname{rk} B \leq 1)$

physical interpretation:

gauge group is Higgsed

if $B = \tilde{B} = 0$, $rk M = k$

$\longrightarrow$ $SU(N_c)$ is broken to $SU(N_c - k)$ with $N_f - k$ flavors

Flat directions of magnetic theory

For non-zero M

$\longrightarrow$ mass term for q, $\tilde{q}$ by superpotential $W = M^i_{\tilde{i}} q_i \tilde{q}^{\tilde{i}}$

If $rk(M) = r$ $\longrightarrow$ r flavors of dual quarks acquire mass, remaining $N_f - r$ quarks are mass-less

1) $r \leq N_c - 2$

F- and D-term allow expectation values for q <u>or</u> $\tilde{q}$ (but not for both) with equal eigenvalues $\longrightarrow$ $SU(N_f - N_c)$ is completely broken

($B \neq 0$, $\tilde{B} = 0$ <u>or</u> $B = 0$, $\tilde{B} \neq 0$)

2) $r \geq N_c - 1$

Use flavor symmetry to bring M to the form

$$M = \begin{pmatrix} \hat{M} & 0 \\ 0 & M_0 \end{pmatrix}$$

with M_0 a square matrix with $N_c - 1$ rows and rank $N_c - 1$.

Consider a flat direction with

$$\text{eigenvalues}(M_0) \gg \text{eigenvalues}\underbrace{(\hat{M})}_{(N_f - N_c + 1)\text{-dim}}$$

$\rightarrow$ integrate out heavy q's :

low energy theory has gauge group $SU(N_f - N_c)$ with $N_f - N_c + 1$ quarks and scale

$$e^{-S_{inst}} = e^{-g^{-2}} = \Lambda_L^{2(N_f - N_c) - 1} = \frac{\det M_0}{\Lambda^{N_c - 1}} \Lambda^{3(N_f - N_c) - N_f}$$

$$\left[\underbrace{3(N_f - N_c) - (N_f - N_c + 1)}_{\text{comes from } \beta\text{-function}} = 2(N_f - N_c) - 1 \right]$$

appears in effective superpotential W_{eff}

At low energies this theory confines
→ use gauge invariant observables:

$$N_a^{\tilde{a}} = \tilde{q}^{\tilde{a}} q_a$$
$$b^a = q^{N_f - N_c}$$
$$\tilde{b}_{\tilde{a}} = \tilde{q}^{N_f - N_c}$$

with $a, \tilde{a} = 1, \ldots, N_f - N_c + 1$ with superpotential

$$\mathrm{Tr}\, \hat{M} N + \frac{1}{\Lambda^{2N_f - 4N_c + 1} \det M_0} (\tilde{b} N b - \det N)$$

flat directions:

$$N = 0$$
$$B\tilde{B} = - \frac{b\tilde{b}}{\Lambda^{2N_f - 4N_c + 1}} = \det M_0 \hat{M}$$

→ precisely match flat directions of electric theory!

But different interpretation:

- electric theory was Higgsed by $\langle Q \rangle \neq 0$
- magnetic theory is confining (strongly coupled)

→ dual variables are magnetic monopoles of the original ones!